

Corrected Analytical Solution of the Generalized Woods-Saxon Potential for Arbitrary ℓ States

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(Published in Phys. Scr. **90** (2015) 015302)

Abstract

The bound state solution of the radial Schrödinger equation with the generalized Woods-Saxon potential is carefully examined by using the Pekeris approximation for arbitrary ℓ states. The energy eigenvalues and the corresponding eigenfunctions are analytically obtained for different n and ℓ quantum numbers. The obtained closed forms are applied to calculate the single particle energy levels of neutron orbiting around ^{56}Fe nucleus in order to check consistency between the analytical and Gamow code results. The analytical results are in good agreement with the results obtained by Gamow code for $\ell = 0$.

PACS numbers: 03.65.Ge, 34.20.Cf, 34.20.Gj

Keywords: Woods-Saxon potential, Eigenvalues and eigenfunctions, analytical solution, Gamow code

I. INTRODUCTION

The Woods-Saxon potential was firstly proposed by R.D. Woods and D.S. Saxon in order to explain the elastic scattering of 20 MeV protons by medium and heavy nuclei approximately sixty years ago[1]. Thereafter the Woods-Saxon potential has taken a great deal of interest over the years and has been one of the most useful model to determine the single particle energy levels of nuclei[2–4] and the nucleus-nucleus interactions[5–7]. The modified version of the Woods-Saxon potential consists of the Woods-Saxon and its derivative called the Woods-Saxon surface potential and is given by[8–10],

$$V(r) = -\frac{V_0}{1 + e^{\frac{r-R}{a}}} - \frac{W_0 e^{\frac{r-R}{a}}}{(1 + e^{\frac{r-R}{a}})^2}, \quad (1)$$

where V_0 and W_0 represent the depths of the potential well. R and a are the radius of the potential and the width of the surface diffuseness, respectively. In Fig.1, a form of the generalized Woods-Saxon potential (GWS) versus the internuclear distance is shown for the given potential parameters with several W_0 values. The surface term in the generalized Woods-Saxon potential induces an extra potential pocket, especially, at the surface region of the potential, and this pocket is very important in order to explain the elastic scattering of some nuclear reactions[5, 6]. Moreover, the Woods-Saxon surface potential induces a potential barrier for $W_0 < 0$ so that it could be used in explaining of the resonant states (quasi-bound states) in nuclei. There are some special cases of the generalized Woods-Saxon potential: The GWS potential is reduced to the standard Woods-Saxon form for $W_0 = 0$ and the square well potential for $W_0 = 0$ and $a \rightarrow 0$ [11]. Furthermore the GWS potential is reduced to the Rosen-Morse potential[12] for $R = 0$ [13].

The relativistic treatment of a Dirac particle in the Woods-Saxon potential field is examined in three dimensions for $\ell = 0$ [14]. Moreover, the transmission coefficient and bound state solutions of one dimensional Woods-Saxon potential are analytically studied[15]. The Klein-Gordon equation in the presence of a spatially one-dimensional Woods-Saxon potential is also examined. The scattering state solutions are obtained in terms of hypergeometric functions and the condition for the existence of transmission resonances is derived[16]. Furthermore s-wave solution of the Dirac equation for a particle moving in the spherically symmetric Woods-Saxon potential under the conditions of the exact spin and the pseudospin symmetry limit is examined[17] and is discussed in Refs.[18, 19].

It is known that the exact analytical solutions of the wave equations (Schrödinger, Dirac, etc.) are very important since a closed form of the wave function is more convenient than the wave function obtained by numerical calculation in explaining the behavior of the system under consideration. Unfortunately, there are few potentials such as harmonic oscillator, Coulomb and Kratzer potentials *etc.*[11] which have the exact analytical solution with centrifugal term. In literature, there are some effort about obtaining the approximate analytic solutions of the wave equations in terms of the $\ell \neq 0$ case: The most widely used approximation is introduced by Pekeris[20] for the exponential-type potential so that this approximation is based on the expansion of the centrifugal barrier in a series of exponentials depending on the internuclear distance up to second order. Greene and Aldrich[21] proposed another approximation for the centrifugal term $1/r^2 \approx \delta^2 e^{\delta r} / (1 - e^{\delta r})^2$. However, this approximation is valid only for small values of the screening parameter δ [22].

Recently, the GWS potential has been examined by using the Nikiforov-Uvarov (NU) method[23] for $\ell = 0$ state[10]. However, in the paper authors have obtained the energy eigenvalue equation as R independent due to $r - R = r$ transformation so that the potential is reduced to the Rosen-Morse potential[13]. It has been noted that the analytical and the numerical results are inconsistent for the GWS potential with $\ell = 0$ state[13]. Similar results with Ref.[10] can be found in Refs.[24–27] for the relativistic or non-relativistic wave equations. In Ref.[24], the approximate analytical solution of the Schrödinger equation for the standard Woods-Saxon potential is obtained for any ℓ state by using the NU method. In Ref.[25], the GWS potential is examined for the Klein-Gordon and Schrödinger equations. In Ref.[26, 27] the Woods-Saxon potential has been analyzed for both the radial Schrödinger and Klein-Gordon equations by using the Pekeris approximation. The authors in Refs.[26, 27] have used $z(r) = \frac{1}{1 + e^{\frac{r-R}{a}}}$ transformation and have obtained R dependent eigenvalue equation by using the Nikiforov-Uvarov method[26, 27]. However, we have realized that the Nikiforov-Uvarov method can not take into account the boundary condition correctly since the Woods-Saxon potential has different character close to $r=R$. Therefore, in this article, we have carefully examined the radial Schrödinger equation with the generalized Woods-Saxon potential by using the Pekeris approximation in terms of the correct boundary conditions. In the next section, we present the calculation procedure. Then, in section III is devoted to the summary and conclusion.

II. THE ENERGY EIGENVALUES AND EIGENFUNCTIONS

The generalized Woods-Saxon potential or special forms of it are very useful in order to describe the interactions between the systems, especially, in nuclear physics. In order to explain the single particle energy levels or elastic scattering of nuclei, the Woods-Saxon potential is generally used. Since the interactions usually occur at the surface region of nuclei for both bound and continuum states, the form of the potential at the surface is crucially important. Therefore the surface term in Eq.(1) would be a very convenient model in order to calculate the single particle energy levels of nuclei. When we consider two-body system with the reduced mass μ moving under the generalized Woods-Saxon potential, the effective potential is,

$$V_{eff}(r) = V(r) + V_\ell(r) = -\frac{V_0}{1 + e^{\frac{r-R}{a}}} - \frac{W_0 e^{\frac{r-R}{a}}}{(1 + e^{\frac{r-R}{a}})^2} + \frac{\ell(\ell+1)\hbar^2}{2\mu r^2}, \quad (2)$$

where $\mu = \frac{m_n m_A}{m_n + m_A}$. m_n and m_A are the atomic mass of the neutron and the core nucleus respectively. There is no analytical solution of Eq.(2) due to polynomial form of the centrifugal barrier term. Therefore, we have to use an approximation for the centrifugal term similar to other authors[26]. In literature there are few approximation procedure[20, 21]. One of them is the Pekeris approximation[20] based on an expansion of the centrifugal barrier depending on the internuclear separation up to second order[26].

The quasi-analytical solution of the effective potential in Eq.(2) with the Pekeris approximation [20] can be obtained within the framework of the Nikiforov-Uvarov (NU) or asymptotic iteration (AIM) methods as follows,

$$n_r(n_r+1) - \beta^2 - \gamma_2^2 + (1+2n_r)\varepsilon + 2\varepsilon^2 + (1+2n_r+2\varepsilon)\sqrt{\varepsilon^2 + \gamma_1^2 - \beta^2} = 0, \quad n_r = 0, 1, 2, \dots \quad (3)$$

with the following definitions:

$$-\varepsilon^2 = \frac{2\mu a^2(E - \delta C_0)}{\hbar^2}, \quad \beta^2 = \frac{2\mu a^2(V_0 - \delta C_1)}{\hbar^2}, \quad \gamma_1^2 = \frac{2\mu a^2 \delta C_2}{\hbar^2}, \quad \gamma_2^2 = \frac{2\mu a^2 W_0}{\hbar^2}, \quad (4)$$

$$C_0 = 1 - \frac{4}{\alpha} + \frac{12}{\alpha^2}, \quad C_1 = \frac{8}{\alpha} - \frac{48}{\alpha^2}, \quad C_2 = \frac{48}{\alpha^2}, \quad \alpha = \frac{R}{a}, \quad \delta = \frac{\ell(\ell+1)\hbar^2}{2\mu R^2}.$$

In Ref.[13], the consistency of the analytical results of Ref.[10] with the Gamow code[31] has been checked by calculating the single particle energy levels of the neutron moving around the ^{56}Fe nucleus for the given potential parameters. As can be seen in Ref.[13], the results are inconsistent with the numerical calculations. We have also confirmed that there are

inconsistencies between Eq.(3) and the Gamow code results for $\ell = 0$. In Eq.(3), if one uses $W_0 = 0$, the results of Ref.[26] can be obtained for arbitrary ℓ states. Furthermore if we take $\ell = 0$ in Eq.(3), we get the results of Ref.[10]. Therefore we should say that Eq.(3) determined by the NU method is incorrect. If we used any analytical solution methods such as the asymptotic iteration method (AIM)[28], the supersymmetry (SUSY)[12], *etc.* to solve the corresponding equations with the generalized Woods-Saxon potential, we would find same results in Eq.(3). In literature, there are similar calculations for the analytical solution of the generalized Woods-Saxon potential with the relativistic or non-relativistic wave equations by using the analytical methods[29]. The origin of the problem is due to the boundary conditions so that the analytical methods cannot take into account them correctly since the Woods-Saxon potential has different characteristic neighborhood $r=R$. Therefore we carefully have to examine the boundary conditions. In order to get the asymptotic behavior of the wave function $u_{n\ell}(z)$, we can use the Nikiforov-Uvarov method[23] and easily get $\phi(z) = z^\varepsilon(1-z)^{\sqrt{\varepsilon^2-\beta^2+\gamma_1^2}}$. As a result, the asymptotic solution of the wave function is,

$$u_{n\ell}(z) = z^\varepsilon(1-z)^\eta f_{n\ell}(z), \quad (5)$$

where $z = \frac{1}{1+e^{\frac{r-R}{a}}}$ and $\eta^2 = \varepsilon^2 - \beta^2 + \gamma_1^2$. The wave function in Eq.(5) satisfies the boundary conditions, *i.e.*, $u_{n\ell}(r \rightarrow 0, z \rightarrow 1) \rightarrow 0$ and $u_{n\ell}(r \rightarrow \infty, z \rightarrow 0) \rightarrow 0$. The Schrödinger equation becomes for Eq.(5),

$$z(1-z)\frac{d^2 f_{n\ell}(z)}{dz^2} + [1+2\varepsilon - (2+2\varepsilon+2\eta)z]\frac{df_{n\ell}(z)}{dz} - [-(\gamma_1^2+\gamma_2^2)+\varepsilon+\eta+(\varepsilon+\eta)^2]f_{n\ell}(z) = 0. \quad (6)$$

It is known that the hypergeometric equation[30] is defined as

$$z(1-z)\frac{d^2 w(z)}{dz^2} + [c - (a+b+1)z]\frac{dw(z)}{dz} - abw(z) = 0, \quad (7)$$

and one of the solutions is $w(z) = {}_2F_1(a, b; c; z)$ [30]. In order to get a, b, c parameters we compare Eq.(6) with Eq.(7) and find,

$$\begin{aligned} a &= \frac{1}{2}(1 \mp \sqrt{1+4\gamma_1^2+4\gamma_2^2+2\varepsilon+2\eta}), \\ b &= \frac{1}{2}(1 \pm \sqrt{1+4\gamma_1^2+4\gamma_2^2+2\varepsilon+2\eta}), \\ c &= 1+2\varepsilon. \end{aligned} \quad (8)$$

Consequently we have $u_{n\ell}(z) = z^\varepsilon(1-z)^{\eta_2} {}_2F_1(a, b; c; z)$. To study in the vicinity of $z = 1$,

we use the relation[11],

$${}_2F_1(a, b; c; y) = \frac{\Gamma(c)\Gamma(c-a-b)}{\Gamma(c-a)\Gamma(c-b)} {}_2F_1(a, b; a+b-c+1; 1-y) + \frac{\Gamma(c)\Gamma(a+b-c)}{\Gamma(a)\Gamma(b)} (1-y)^{c-a-b} {}_2F_1(c-a, c-b; c-a-b+1; 1-y). \quad (9)$$

${}_2F_1(a, b; c; 0)$ is equal 1[30]. Therefore, by using Eq.(9) and the boundary condition $u_{n\ell}(r \rightarrow 0, z \rightarrow 1) = 0$, we get

$$\frac{\Gamma(a+b-c)}{\Gamma(c-a-b)} \frac{\Gamma(c-a)}{\Gamma(b)} \frac{\Gamma(c-b)}{\Gamma(a)} (1 + e^{R/a})^{2\eta} = -1, \quad (10)$$

where $\eta = i\lambda$ and $\lambda = \sqrt{\beta^2 - \gamma_1^2 - \varepsilon^2}$. Evaluating Eq.(10) for the given a,b,c parameters in Eq.(8), we obtain

$$\frac{\Gamma(2i\lambda)}{\Gamma(-2i\lambda)} \frac{\Gamma(\frac{1}{2} + \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon - i\eta)}{\Gamma(\frac{1}{2} - \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon + i\eta)} \frac{\Gamma(\frac{1}{2} - \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon - i\eta)}{\Gamma(\frac{1}{2} + \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon + i\eta)} \times (1 + e^{R/a})^{2\eta} = -1. \quad (11)$$

In Eq.(11), $e^{R/a} \gg 1$ for a given realistic parameters. Therefore we can use an approximation $1 + e^{R/a} \approx e^{R/a}$ with small errors in the eigenvalues. We can easily get the following equation by using $e^{-2i\arg\Gamma(z)} = \frac{\Gamma^*(z)}{\Gamma(z)}$ relation,

$$e^{2i\left[\arg\Gamma(2i\lambda) - \arg\Gamma(\frac{1}{2} + \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon + i\eta) - \arg\Gamma(\frac{1}{2} - \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon + i\eta) + \frac{R\lambda}{a}\right]} = -1. \quad (12)$$

Therefore, the corrected energy eigenvalue equation in a closed form becomes,

$$\arg\Gamma(2i\lambda) - \arg\Gamma(\frac{1}{2} + \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon + i\eta) - \arg\Gamma(\frac{1}{2} - \frac{1}{2}\sqrt{1+4\gamma_1^2+4\gamma_2^2} + \varepsilon + i\eta) + \frac{R\lambda}{a} = (n_r + \frac{1}{2})\pi, \quad n_r = 0, \pm 1, \pm 2, \dots, \quad (13)$$

where n_r is the radial node number. The quantum number is $n = n_r + 1$. In order to test the accuracy of Eq.(13), we calculate the single particle energy levels of neutron rotating around ^{56}Fe nucleus by using the potential parameters given in Ref.[10]. The Woods-Saxon potential parameters are $V_0 = 40.5 + 0.13A = 47.78$ MeV, $R = 4.9162$ fm and $a = 0.6$ fm. Here A is the atomic mass number of ^{56}Fe nucleus. The reduced mass consists of neutron mass $m_n = 1.00866u$, and ^{56}Fe core mass $m_A = 56u$. In Table I, we show agreement between the energy eigenvalue equation given by Eq.(13) and the numerical calculation obtained by Gamow code[31] for the neutron plus ^{56}Fe nucleus system with several n_r quantum numbers and W_0 parameters. There are small inaccuracy in the analytic and numeric results since

we have made the approximation in Eq.(11). It might be seen that neutron is unbound for $n_r = 3, W_0 = 0$ and Eq.(13) gives $E_{n_r\ell} = 62.9775\text{MeV}$, but this finding is not acceptable as physically and is only a mathematical result which satisfies Eq.(13). There are similar situations for $n_r = 3, W_0 = 50\text{MeV}$; $n_r = 3, W_0 = 100\text{MeV}$; and $n_r = 2, 3, W_0 = -50\text{MeV}$. However there are the quasi-bound states (resonant states) for $n_r = 2, W_0 = -100\text{ MeV}$ since the nuclear potential has a potential barrier inducing the resonant states in Fig.1. Our result for $n_r = 2, W_0 = -100\text{ MeV}$ is physically incorrect. In order to calculate the energy eigenvalues of the resonant states, the Complex Scaling Method(CSM) can be used[32]. Another interesting point is that we cannot calculate the bound state energy eigenvalues by using Eq.(13) for $n_r = 2, 3, W_0 = 100\text{MeV}$ since the right and left sides of Eq.(13) do not intersect in real line. It should be noted that the ℓ -state solution of the generalized Woods-Saxon potential in terms of the Pekeris approximation is valid only for small α values.

The radial wave function corresponding to the eigenvalue equation Eq.(13) in terms of Eq.(4) and Eq.(8) can be written in a closed form as follows,

$$u_{n\ell}(r) = N \left(\frac{1}{1 + e^{\frac{r-R}{a}}} \right)^\varepsilon \left(1 - \frac{1}{1 + e^{\frac{r-R}{a}}} \right)^\eta \quad (14)$$

$$\times {}_2F_1\left(\frac{1}{2} - \frac{1}{2}\sqrt{1 + 4\gamma_1^2 + 4\gamma_2^2} + \varepsilon + \eta, \frac{1}{2} + \frac{1}{2}\sqrt{1 + 4\gamma_1^2 + 4\gamma_2^2} + \varepsilon + \eta; 1 + 2\varepsilon; \frac{1}{1 + e^{\frac{r-R}{a}}}\right),$$

where N is the normalization constant. The unnormalized wave function fulfilling the boundary conditions is shown in Fig.2 for several radial quantum numbers n .

III. CONCLUSION

We have studied the approximate analytical solution of the Schrödinger equation in the presence of the generalized Woods-Saxon potential by using the Pekeris approximation and the properties of the Hypergeometric functions for any ℓ states. We have seen that the Nikiforov-Uvarov method cannot take into account the correct boundary conditions for the generalized Woods-Saxon potential. Therefore we have carefully examined the asymptotic behavior of the wave function of the generalized Woods-Saxon potential and have obtained the corrected eigenvalue equation and the corresponding eigenfunction in the closed form for any ℓ states. We have also calculated the single particle energy level of neutron rotating around ^{56}Fe nucleus for the given potential parameters in order to check the consistency between the analytical and Gamow code results. We have shown that the obtained analytical

results in this study are in good agreement with the results obtained by the Gamow code for $\ell=0$ state. The resonant state solutions of the generalized Woods-Saxon potential in a closed form are in progress.

ACKNOWLEDGMENTS

Authors would like to thank TÜBİTAK and Akdeniz University for the their financial supports as well as Dr. A. Soylu for useful comments on the manuscript.

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$W_0(\text{MeV})$	n_r	$E_{n_r}^{\text{Analytical}}(\text{MeV})$	$E_{n_r}^{\text{Gamow}}(\text{MeV})$
0	0	-38.3004	-38.3002
0	1	-18.2254	-18.2227
0	2	-0.2678	-0.2663
0	3	62.9775	unbound
50	0	-41.1965	-41.1964
50	1	-23.8789	-23.8788
50	2	-3.6472	-3.6471
50	3	52.0232	unbound
100	0	-45.4453	-45.4446
100	1	-29.1659	-29.1642
100	2	undetermined	-7.8143
100	3	undetermined	unbound
-50	0	-36.2136	-36.2168
-50	1	-12.8469	-12.8504
-50	2	18.5701	unbound
-50	3	64.2816	unbound
-100	0	-34.5902	-34.5956
-100	1	-8.0843	-8.0902
-100	2	19.2098	17.74+i(-8.36)

TABLE I. Comparison of the analytical and numerical results for the single particle energy levels of neutron orbiting around ^{56}Fe nucleus with several W_0 potential depth and n_r quantum numbers for $\ell = 0$. The potential parameters are $V_0 = 40.5 + 0.13A = 47.78$ MeV, $R = 4.9162$ fm and $a = 0.6$ fm. The reduced mass consists of neutron mass $m_n = 1.00866u$, and ^{56}Fe core mass which is $m_A = 56u$ and its value is $\mu = 0.990814u$.

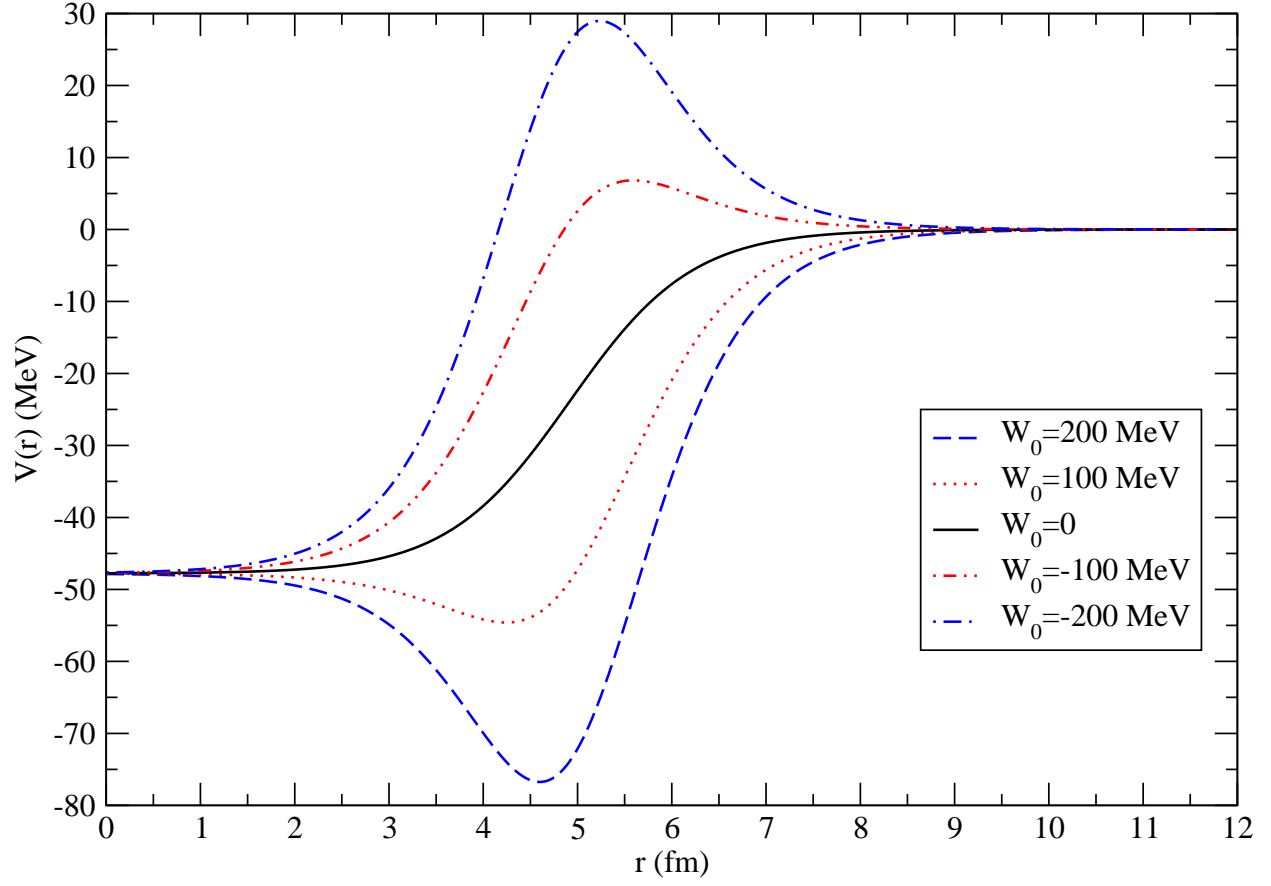


FIG. 1. Variation of the generalized Woods-Saxon potential as a function of the internuclear distance and several W_0 values for $V_0=40.5+0.13A$ MeV, $R=4.9162$ fm and $a=0.6$ fm.

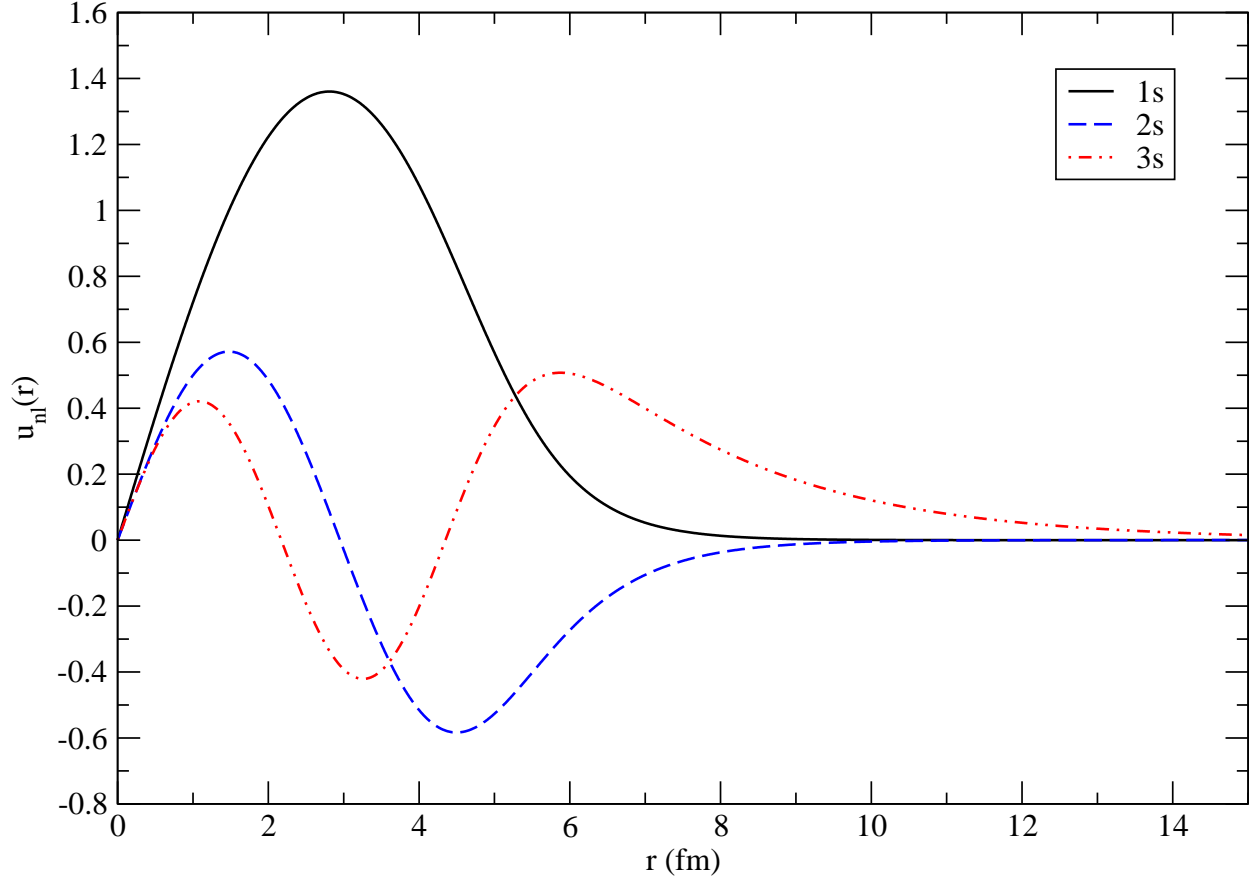


FIG. 2. The unnormalized wave function of the generalized Woods-Saxon potential for $V_0=40.5+0.13A$ MeV, $W_0=50$ MeV, $R=4.9162$ fm and $a=0.6$ fm potential parameters with several n quantum numbers.